11.2

DOES $\sum_{i=1}^{\infty} ar^{i-1}$ CONV/DIV ? (ASSUME $a \neq 0$)

IE. $\lim_{n\to\infty} S_n$ EXIST? $\nearrow$ YES $\rightarrow$ CONV ; $\searrow$ NO $\rightarrow$ DIV

$$S_n = \sum_{i=1}^{n} ar^{i-1} = a + ar + ar^2 + ar^3 + \ldots + ar^{n-1}$$

IF $r = 1 \longrightarrow \sum_{i=1}^{\infty} ar^{i-1}$ DIV

$= a + a + a + a + \ldots$

IF $r \neq 1 \longrightarrow S_n = \dfrac{a(1-r^n)}{1-r}$

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty} \frac{a(1-r^n)}{1-r}$$

$$= \frac{a}{1-r} \lim_{n\to\infty} (1-r^n)$$

$$= \frac{a}{1-r} \left(1 - \lim_{n\to\infty} r^n\right)$$

$\lim_{n\to\infty} r^n = 0$ IF $|r| < 1$

DNE IF $|r| > 1$ $(=\pm\infty)$

DNE IF $r = -1$

IF $|r| < 1$, THEN $\sum_{i=1}^{\infty} ar^{i-1}$ CONV TO $\dfrac{a}{1-r}$

OTHERWISE, $\sum_{i=1}^{\infty} ar^{i-1}$ DIV

IE. $|r| \geq 1$

DOES $\sum_{i=1}^{\infty} 5^{i+2} 3^{1-2i}$ CONV/DIV? IF IT CONV, TO WHAT?

$\frac{a}{1-r}$

$$= \sum_{i=1}^{\infty} \frac{125}{3}\left(\frac{5}{9}\right)^{i-1}$$

$a = \frac{125}{3}$

$r = \frac{5}{9}$

$|r|<1 \longrightarrow \sum$ CONV

CONVERGES

$$= \frac{\frac{125}{\cancel{3}}}{1-\frac{5}{9}} \cdot \frac{\cancel{9}^{3}}{9}$$

$$= \frac{375}{9-5} = \frac{375}{4}$$

$$5^{i+2}\, 3^{1-2i}$$
$$= 5^i 5^2 3^1 \cdot 3^{-2i}$$
$$= 75 \cdot 5^i \left(3^{-2}\right)^i$$
$$= 75 \cdot 5^i \left(\tfrac{1}{9}\right)^i$$
$$= 75\left(\tfrac{5}{9}\right)^i$$
$$= \overset{25}{\cancel{75}}\left(\tfrac{5}{9}\right)^{i-1}\frac{5}{\cancel{9}_{3}}$$
$$= \frac{125}{3}\left(\frac{5}{9}\right)^{i-1}$$

OR

$$\{5^{i+2}\, 3^{1-2i}\}$$

$$= \{5^3 3^{-1}, 5^4 3^{-3}, 5^5 3^{-5}, 5^6 3^{-7}, \ldots\}$$

$*5\cdot 3^{-2}$ $\quad *5\cdot 3^{-2}$ $\quad *5\cdot 3^{-2}$

$a = \frac{125}{3}$ $\quad r = \frac{5}{9}$

$$\sum_{i=0}^{\infty}(x-1)^i = 1 + (x-1) + (x-1)^2 + (x-1)^3 + \cdots$$

$$1 \xrightarrow{*(x-1)} (x-1) \xrightarrow{*(x-1)} (x-1)^2 \xrightarrow{*(x-1)} (x-1)^3$$

$a = 1$

$r = x-1$

$\sum$ CONV IF $|r| < 1$

$|x-1| < 1$

$-1 < x-1 < 1$

$|y| < 3$

$-3 < y < 3$

$\sum_{i=0}^{\infty}(x-1)^i$ CONV IF $0 < x < 2$

DIV IF $x \leq 0$ OR $x \geq 2$

IF $0 < x < 2$, $\sum_{i=0}^{\infty}(x-1)^i = \frac{1}{1-(x-1)} = \frac{1}{2-x}$

LAWS OF INFINITE SERIES (LINEARITY)

IF $\sum_{i=1}^{\infty} a_i$ CONV, THEN $\sum_{i=1}^{\infty} ca_i = c\sum_{i=1}^{\infty} a_i$ FOR ALL $c \in \mathbb{R}$

IF $\sum_{i=1}^{\infty} a_i$ AND $\sum_{i=1}^{\infty} b_i$ CONV, THEN $\sum_{i=1}^{\infty}(a_i \pm b_i) = \sum_{i=1}^{\infty} a_i \pm \sum_{i=1}^{\infty} b_i$

$$\sum_{i=1}^{\infty}\left(\frac{1}{i(i+2)} - 5^{i+2}3^{1-2i}\right)$$

$$= \sum_{i=1}^{\infty}\frac{1}{i(i+2)} - \sum_{i=1}^{\infty} 5^{i+2}3^{1-2i}$$ IF BOTH $\sum$ CONV ✓

$$= \sum_{i=1}^{\infty}\frac{1}{2}\cdot\frac{2}{i(i+2)} - \frac{375}{4}$$

$$= \frac{1}{2}\cdot\sum_{i=1}^{\infty}\frac{2}{i(i+2)} - \frac{375}{4}$$

$$= \frac{1}{2}\cdot\frac{3}{2} - \frac{375}{4}$$

FROM YESTERDAY

$$= \frac{3}{4} - \frac{375}{4}$$

$$= -\frac{372}{4}$$

$$= -93$$

HARMONIC SERIES

$$\sum_{i=1}^{\infty} \frac{1}{i} = 1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \frac{1}{5} + \cdots$$

NOTE $\lim_{n\to\infty} \frac{1}{n} = 0$

IE. $\boxed{\left\{\frac{1}{n}\right\} \text{ CONV TO } 0}$

$$S_n = \sum_{i=1}^{n} \frac{1}{i} = 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n}$$

$$S_2 = 1 + \frac{1}{2}$$

$$S_4 = 1 + \frac{1}{2} + \left(\frac{1}{3} + \frac{1}{4}\right) > 1 + \frac{1}{2} + \left(\frac{1}{4} + \frac{1}{4}\right) = 1 + \frac{1}{2} + \frac{1}{2} = 1 + \frac{2}{2}$$

$$S_8 = 1 + \frac{1}{2} + \left(\frac{1}{3} + \frac{1}{4}\right) + \left(\frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{8}\right) > 1 + \frac{1}{2} + \left(\frac{1}{4} + \frac{1}{4}\right) + \left(\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}\right)$$

$$= 1 + \frac{1}{2} + \frac{1}{2} + \frac{1}{2} = 1 + \frac{3}{2}$$

NOTE:

$$S_{2^1} > 1 + \frac{1}{2}$$

$$S_{2^2} > 1 + \frac{2}{2}$$

$$S_{2^3} > 1 + \frac{3}{2}$$

$$S_{2^k} > 1 + \frac{k}{2} \qquad 1 + \frac{k}{2} \longrightarrow \infty \text{ AS } k \to \infty$$

SO WE CAN MAKE S_n BE AS LARGE AS WE WANT BY LETTING n BE SUFFICIENTLY LARGE

IE. $S_n \to \infty$ IE. $\lim_{n\to\infty} S_n = \infty$ IE. $\boxed{\sum_{i=1}^{\infty} \frac{1}{i} \text{ DIV}}$